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ORIGINAL ARTICLE

Fuzzy Logic Based on Belief and Disbelief Membership Functions

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Abstract Many theories are developed based on probability to deal with incomplete information. The fuzzy logic deals with belief rather than likelihood (probability). Zadeh first defined fuzzy set as a single membership function. The two fold fuzzy sets with two membership functions will give more evidence than a single membership one. Therefore there is need of fuzzy logic with two membership functions. In this paper, The fuzzy set is defined with two membership functions "Belief" and "Disbelief ". The fuzzy inference and fuzzy reasoning are studied for "a two fold fuzzy set". The fuzzy certainty factor (FCF) is defined as a single membership function by taking difference between "Belief" and "Disbelief ". The quantification of fuzzy truth variables are studied for "a two fold fuzzy set". The medical expert system shell EMYCIN is given as an application of "a two fold fuzzy set".

Keywords Fuzzy sets · Fuzzy logic and fuzzy reasoning with two fold fuzzy set · Fuzzy certainty factor· Fuzzy truth variables· Medical expert system· EMYCIN

1. Introduction

There are many theories proposed to deal with incomplete information like fuzzy logic, fuzzy probability, probability theory, Dempster-Shafer theory, certainty fac-

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tor, many-valued Logic, possibility theory, plausibility logic, non-monotonic logic, etc. The fuzzy logic[20] deals with belief rather than likelihood (probability). Zadeh defined fuzzy set with a single membership function. The fuzzy set with a two membership functions will give more evidence than a single membership function. For instance "Rama has Headache". In this fuzzy proposition, belief and disbelief are to be considered for better judgement. The fuzzy set "Headache" is necessary to study with a two membership functions $\mu^{Belief}_{Headache}(x)$ and $\mu^{Disbelief}_{Headache}(x)$. The fuzzy set with two membership functions is defined with "True"and "False"[6] and extend to Zadeh fuzzy logic [20]. In MYCIN [1], the incomplete information is defined with two functions MB[h,e] and MD[h,e], where "h" hypothesis and "e" is evidence. MB[h,e] and MD[h,e] are probabilities. The certainty factor is defined as difference between functions MD[h,e] and MD[h,e]. It is possible to define MB[h,e] and MD[h,e] as fuzzy sets and fuzzy certainty factor as difference between these fuzzy sets.

In the following, fuzzy logic and fuzzy reasoning is given briefly for a single fuzzy membership function. The fuzzy logic and reasoning are studied for "two fold fuzzy set"with membership functions $\mu^{Belief}_{Headache}(x)$ and $\mu^{Disbelief}_{Headache}(x)$. The fuzzy certainty factor is studied as difference between two fold fuzzy sets "belief"and "disbelief". The quantification of fuzzy truth variables are studied for "two fold fuzzy set" and finally medical expert system shell EMYCIN is discussed as an example for "two fold fuzzy sets".

2. The Fuzzy Logic

The fuzzy logic [20] is defined as a model to deal with imprecise, incomplete, vague and inexact information transition. The fuzzy set is a class of objects with a continuum of grades of membership.

Definition 2.1 *Fuzzy set A in a universe of discourse X is defined as its membership function* $\mu_A(x) :\rightarrow [0,1], x \in X$. $A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \mu_A(x_n)/x_n$, where "+" is union.

For example, the fuzzy proposition "x is young" is given as with membership function.

$$\mu_{young}(x) = (1 + (n/30)^2)^{-1}$$
$$= \{0.9/10 + 0.8/15 + 0.69/20 + 0.59/25 + 0.5/30 + 0.42/35 + 0.36/40 + 0.31/45 + 0.26/50\}.$$

The fuzzy proposition "x is not young" is also given as .

$$1 - \mu_{notyoung}(x) = (1 + (n/30)^2)^{-1}$$
$$= \{0.1/10 + 0.2/15 + 0.31/20 + 0.41/25 + 0.5/30 + 0.58/35 + 0.64/40 + 0.69/45 + 0.74/50\},$$

For instance, the fuzziness of "Rama who is 40 years old is YOUNG is 0.64".

The Graphical representation of "young" and "not young" is shown in Fig.1.

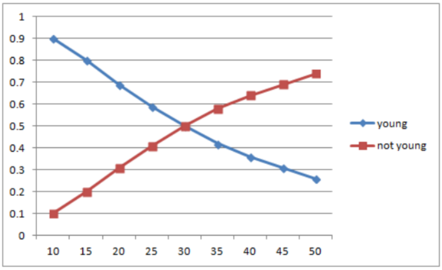

Fig.1 Fuzzy membership function

The fuzzy logic [18, 20] is combination of fuzzy sets by using logical operators. Some of the logical operations on fuzzy sets are given below.

Negation

x is not A

$A' = 1 - \mu_A(x)/x.$

Conjunction

x is A and y is $B \rightarrow$(x,y) is $A \times B$,

$A \times B$= $\min\{\mu_A(x), \mu_B(y)\}/(x, y)$,

if $x = y$,

x is A and x is $B \rightarrow x$ is $A \wedge B$,

$A \wedge B$= $\min\{\mu_A(x), \mu_B(x)\}/x$.

Disjunction

x is A or y is $B \rightarrow (x, y)$ is $A + B$,

$A + B$= $\max\{\mu_A(x), \mu_B(y)\}/(x, y)$,

if $x = y$,

x is A or x is $B \rightarrow x$ is $A \vee B$,

$A \vee B$= $\max\{\mu_A(x), \mu_B(x)\}/x$.

Implication

Zadeh [18] fuzzy conditional inference is given as

if x is A then y is B =$A \rightarrow B = (x, y)$ is $(P \oplus Q)$,

$A \rightarrow B = A \oplus B = \min\{1, 1 - \mu_A(x) + \mu_B(y)\}/(x, y)$.

Mamdani [5] fuzzy conditional inference given as

if x is A then y is B =$A \rightarrow B = AXB = \min\{\mu_A(x), \mu_B(y)\}/(x, y)\}$,

if prior information is not available to the consequent part, the fuzzy conditional inference may be given as

TSK [8] fuzzy conditional inference is given when consequent part is not known as

if x is A then $y = f(x)$ is B,
if x_1 is A_1 and x_2 is $A_2 \cdots$ x_n is A_n, then $y = f(x_1, x_2, \cdots, x_n)$ is B.
Reddy [14] fuzzy conditional inference is considered fuzzy sets A_i instead of variable x_i when consequent part is not known
if x_1 is A_1 and x_2 is A_2 and $\cdots$ and x_n is A_n then y is

$$B = f(A_1, A_2, \cdots, A_n) = \min(A_1, A_2, \cdots, A_n).$$

Zadeh[18] fuzzy conditional inference is given as for
if x is A then y is B else y is $C = A \times B + A' \times C$,

$$A \times B + A' \times C = \min\{\min(\mu_A(x), \mu_B(y)), \min(1 - \mu_A(x)), \mu_C(y))\}/(x, y).$$

Reddy [14] fuzzy conditional inference is given as for
"if x is A then y is B else y is C " may be divided into two clauses "if x is A then y is B " and "if x is not A then y is C ".
if x is A then y is B else y is $C = A \to B = A \oplus B$

$$= \min\{1, 1 - \mu_A(x)) + \mu_B(y)\}/(x, y),$$

if x is not A then y is C else y is $C = A' \to C = A' \oplus C$

$$= \min\{1, \mu_A(x)) + \mu_C(y)\}/(x, y).$$

Composition

if x is A then y is B
x is A_1

---

y is $A_1 o R$

$$A_1 oR = \min\{\mu_{A_1}(x)), \mu_R(x, y)\}/y$$
$$= \min\{\mu_{A_1}(x), \min(1, 1 - \mu_A(x) + \mu_B(y))\}/y$$

Fuzzy truth variables

x is A is τ is given by
x is $\mu_A^{-1}(x)$ o τ, where $\mu_A(x)^{-1}$ inverse of compatibility function of A and τ is truth variable.

For instance, τ may be "true", "very true", "false","more or less false" etc.

Fuzzy quantifiers

The fuzzy propositions may contain quantifiers like "very", "very very" (Concentration), "rather", "more or less" (Diffusion). These fuzzy quantifiers may be eliminated as

Concentration

x is very A
$\mu_{very\,A}(x) = \mu_A(x)^2$.

Diffusion

x is more or less A
$\mu_{more\ or\ less\ A}(x) = \mu_A(x)^{0.5}$.

Fuzzy Reasoning

The fuzzy reasoning is drawing a conclusion from fuzzy propositions [8, 11]. The fuzzy reasoning is given bellow single membership function.

R-1

x is A and y is B
x is A_1

---

y is A_1 o $(A \wedge B)$

R-2

x is A or y is B
x is A_1

---

y is A_1 o $(A \vee B)$

R-3

x and y are B
y and z are B

---

x and z are AoB

R-4

if x is A then y is B
x is A_1

---

y is A_1 o$(A \to B)$

R-5

if x is A then y is B else y is C
x is A_1

---

y is A_1 o $(A \times B + A' \times C)$

R-6

if x is A then y is B else y is C
x is A_1

---

y is A_1 o $(A \to B)$

R-7

if x is not A and y is B else y is C
x is not A_1

---

y is not A_1 o $(A' \to C)$

R-8

if x is A then y is B is τ_1
x is A_1 is τ_2

---

y is A_1 o $(A \to B)$ is $\tau_1 o \tau_2$
where τ_1 and τ_2 are fuzzy truth variables.

3. Generalized Fuzzy Logic

Zadeh defined fuzzy set with a single membership function [20]. The fuzzy set with two fuzzy member functions "Belief" and "Disbelief"will give more evidence than the single fuzzy membership function to deal with incomplete information. In the following "two fold fuzzy set" is defined with "Belief" and "Disbelief"fuzzy membership functions. The fuzzy logic and fuzzy reasoning of single membership function is extended to fuzzy logic with two membership functions "Belief" and "Disbelief".

3.1. The Two Fold Fuzzy Sets

"A two fold fuzzy set" may be defined with two membership functions "Belief" and "Disbelief" for the proposition of type "x is A". The fuzzy set with two membership functions "Belief"and "Disbelief" will give more evidence than the single membership function.

For instance "Rama has Headache ". In this fuzzy proposition, the fuzzy set Headache, the "Headache" may be defined with "Belief" and "Disbelief" to deal the incomplete information.

Definition 3.1 *The "a two fold fuzzy set" $\tilde{A}$ in a universe of discourseX is defined by its membership function $\mu_{\tilde{A}}(x) \to [0,1]$, where $\tilde{A} = \{\mu_A^{Belief}(x), \mu_A^{Disbelief}(x)\}$ and $x \in X\}$,*

$\mu_A^{Belief}(x)$ and $\mu_A^{Disbelief}(x)$ are the fuzzy membership functions of the "a two fold fuzzy set"$\tilde{A}$,

$$\mu_A^{Belief}(x) = \mu_A^{Belief}(x_1)/x_1 + \cdots + \mu_A^{Belief}(x_n)/x_n,$$

$$\mu_A^{Disbelief}(x) = \mu_A^{disbelief}(x_1)/x_1 + \cdots + \mu_A^{Disbelief}(x_n)/x_n, \text{ where "+" is union,}$$

$$\mu_A^{Belief}(x) + \mu_A^{Disbelieff}(x) < 1,$$

$$\mu_A^{Belief}(x) + \mu_A^{Disbelieff}(x) > 1,$$

and

$$\mu_A^{Belief}(x) + \mu_A^{Disbelieff}(x) = 1,$$

are interpreted as redundant, insufficient and sufficient information respectively.

For example "x is young "and young may be given as

$$\text{young} = \{\mu_{young}^{Belief}(x), \mu_{young}^{Disbelief}(x)\},$$

$$\mu_{young}^{Belief}(x) = (1 + (n/30)^2)^{-1})$$
$$= \{0.9/10 + 0.8/15 + 0.69/20 + 0.59/25 + 0.5/30$$
$$+ 0.42/35 + 0.36/40 + 0.31/45 + 0.26/50\},$$

$$\mu_{young}^{Disbelief}(x) = (1 + ((n + 30)/30)^2)^{-1}$$
$$= \{0.36/10 + 0.31/15 + 0.26/20 + 0.23/25 + 0.2/30 + 0.18/35$$
$$+ 0.16/40 + 0.14/45 + 0.12/50\}.$$

For instance. "Rama is young" with fuzziness $\{0.8, 02\}$, where 0.8 is "Belief" and 0.2 is "Disbelief".

The Graphical representation of "Belief" and "Disbelief" of "young" is shown in Fig. 2.

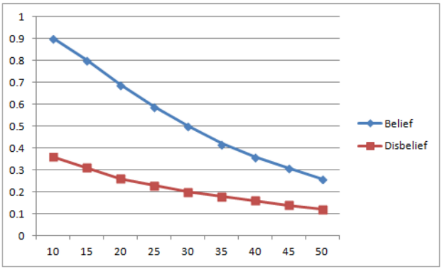

Fig.2 Two fold fuzzy set membership functions

3.2. The Fuzzy Logic with "Two Fold Fuzzy Sets"

A two fold fuzzy set is defined with fuzzy membership functions "Belief " and "Disbelief ". The fuzzy logic is combination of fuzzy sets using logical operators. The fuzzy logic with "two fold fuzzy sets" is combination of "two fold fuzzy sets" using logical operators. The fuzzy logic bases on "two fold fuzzy sets" can be studied similar lines of Zadeh's fuzzy logic.

Some of the logical operations are given below for fuzzy sets with two fold fuzzy membership functions.

$\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets with two fold fuzzy membership functions.

Let tall, weight and more or less weight are two fold fuzzy sets.

$$\tilde{\text{tall}} = \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5,$$
$$0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$$
$$\tilde{\text{weight}} = \{0.8/x_1 + 0.7/x_2 + 0.5/x_3 + 0.3/x_4 + 0.2/x_5,$$
$$0.2/x_1 + 0.2/x_2 + 0.1/x_3 + 0.1/x_4 + 1/x_5\}$$
$$\text{more or less } \tilde{\text{weight}}= \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.5/x_4 + 0.4/x_5,$$
$$0.4/x_1 + 0.4/x_2 + .3/x_3 + .3/x_4 + 0.3/x_5\}.$$

Negation

x is not $\tilde{A}$

$$\tilde{A}'(x) = \{1 - \mu_A^{Belief}(x), 1 - \mu_A^{Disbelief}(x)\}/x$$

x is not $\tilde{tall}$
$\tilde{tall} = \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5,$
$0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$
$1 - \tilde{tall} = \{0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.6/x_4 + 0.8/x_5,$
$0.5/x_1 + 0.6/x_2 + 0.7/x_3 + 0.8/x_4 + 0.9/x_5\}.$

Disjunction
x is $\tilde{A}$ or y is $\tilde{B}$
$\tilde{A} + \tilde{B} = \{\max(\mu_A^{Belief}(x), \mu_B^{Belief}(y)), \max(\mu_A^{Disbelief}(x), \mu_B^{Disbelief}(y))\}/(x, y),$
if $x = y$
$\tilde{A} \vee \tilde{B} = \{\max(\mu_A^{Belief}(x), \mu_B^{Belief}(x)), \max(\mu_A^{Disbelief}(x), \mu_B^{Disbelief}(x))\}/x,$
x is $\tilde{tall}$ or x is $\tilde{weight}$,
$\tilde{tall} \vee \tilde{weight} = \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.6/x_4 + 0.5/x_5,$
$0.4/x_1 + 0.3/x_2 + 0.2/x_3 + .1/x_4 + .1/x_5\}.$

Conjunction
x is $\tilde{A}$ and y is $\tilde{B}$
$\tilde{A} \times \tilde{B} = \{\min(\mu_A^{Belief}(x), \mu_B^{Belief}(y)), \min(\mu_A^{Disbelief}(x), \mu_B^{Disbelief}(y))\}/(x, y),$
if $x = y$
$\tilde{A} \wedge \tilde{B} = \{\min(\mu_A^{Belief}(x), \mu_B^{Belief}(x)), \min(\mu_A^{Disbelief}(x), \mu_B^{Disbelief}(x))\}/x,$
x is $\tilde{tall}$ and x is $\tilde{weight}$,
$\tilde{tall} \wedge \tilde{weight} = \{0.8/x_1 + 0.7/x_2 + 0.5/x_3 + 0.2/x_4 + 0.2/x_5,$
$0.1/x_1 + 0.1/x_2 + 0.1/x_3 + 0.1/x_4 + 0.1/x_5\}.$

Implication
Zadeh [18] fuzzy conditional inference is given as
if x is $\tilde{A}$ then y is $\tilde{B} = \tilde{A} \to \tilde{B} = \tilde{A} \oplus \tilde{B}$,
$=\{\min(1, 1 - \mu_A^{Belief}(x) + \mu_B^{Belief}(y)), \min(1, 1 - \mu_A^{Disbelief}(x) + \mu_B^{Disbelief}(y))\}/(x, y),$
if $x = y$
$= \{\min(1, 1 - \mu_A^{Belief}(x) + \mu_B^{Belief}(x)), \min(1, 1 - \mu_A^{Disbelief}(x) + \mu_B^{Disbelief}(x))\}/x.$
if x is $\tilde{tall}$ then x is $\tilde{weight}$,
$\tilde{tall} \to \tilde{weight} = \{0.9/x_1 + 0.9/x_2 + 0.8/x_3 + 0.9/x_4 + 1/x_5,$
$0.7/x_1 + 0.8/x_2 + 0.9/x_3 + 0.9/x_4 + 1/x_5\}.$
Mamdani [5] fuzzy conditional inference is given as $\tilde{A} \to \tilde{B} = \tilde{A} \times \tilde{B}$
if prior information is not available to the system, the fuzzy conditional inference may be given as
if prior information is not available to the system i.e., the relationship between A and B is not known.

TSK [8] fuzzy conditional inference is given as
if x_1 is $\tilde{A_1}$ and x_2 is $\tilde{A_2}$ and $\cdots$ and x_n is $\tilde{A_n}$ then y=$f(x_1, x_2 \cdots x_n)$ is $\tilde{B}$.
Reddy [13] fuzzy conditional inference is given as,
if x_1 is $\tilde{A_1}$ and x_2 is $\tilde{A_2}$ and $\cdots$ and x_n is $\tilde{A_n}$ then y is $\tilde{B}$ =min$\{\tilde{A_1}, \tilde{A_2}, \cdots, \tilde{A_n}\}$
Zadeh [18] fuzzy conditional inference is given as
if x is $\tilde{A}$ then y is $\tilde{B}$ else y is $\tilde{C} = (\tilde{A} \times \tilde{B} + \tilde{A}' \times \tilde{C},)$ where "+ " is union

if x is $\tilde{tall}$ then x is $\tilde{weight}$ else x is more or less $\tilde{weight}$

$= \{0.8/x_1 + 0.7/x_2 + 0.5/x_3 + 0.5/x_4 + 0.4/x_5, 0.4/x_1 + 0.4/x_2 + 0.3/x_3 + 0.3/x_4 + 0.3/x_5\}.$

Reddy [13] fuzzy conditional inference is given for "if x is $\tilde{A}$ then y is $\tilde{B}$ else y is $\tilde{C}$" as,

if x is $\tilde{A}$ then y is $\tilde{B} = \tilde{A} \to \tilde{B}$

if x is not $\tilde{A}$ then y is $\tilde{C} = \tilde{A}' \to \tilde{C}$

"if x is not $\tilde{tall}$ then x is $\tilde{weight}$ else x is more or less $\tilde{weight}$" is given as,

$\tilde{tall} \to \tilde{weight}$

$=\{0.9/x_1+0.9/x_2+0.8/x_3+0.9/x_4+1/x_5, 0.7/x_1+0.8/x_2+0.8/x_3+0.9/x_4+1/x_5\},$

$\tilde{tall}' \to$ more or less $\tilde{weight}$

$= \{1/x_1+1/x_2+1/x_3+0.9/x_4+0.6/x_5, 0.9/x_1+0.8/x_2+0.6/x_3+0.5/x_4+0.4/x_5\}.$

Composition

if x is $\tilde{A}$ then y is $\tilde{B}$

x is $\tilde{A}_1$

----

y is $\tilde{A}_1$ o $(\tilde{A} \to \tilde{B})$

$$\tilde{A} \text{ o } (\tilde{A} \to \tilde{B}) = \{\min\{\mu_A^{Belief}(x), \min(1, 1-\mu_A^{Belief}(x)+\mu_B^{Belief}(y))\}, \min\{\mu_A^{Diselief}(x), \min(1, 1-\mu_A^{Disbelief}(x)+\mu_B^{Disbelief}(y)\}\}/y,$$

if $x = y$

$$= \{\min\{\mu_A^{Belief}(x), \min(1, 1-\mu_A^{Belief}(x)+\mu_B^{Belief}(x))\}, \min\{\mu_A^{Diselief}(x), \min(1, 1-\mu_A^{Disbelief}(x)+\mu_B^{Disbelief}(x))\}.$$

if x is $\tilde{tall}$ then x is $\tilde{weight}$

x is very $\tilde{tall}$

----

x is very $\tilde{tall}$ o ($\tilde{tall} \to \tilde{weight}$)

very $\tilde{tall}$ o ($\tilde{tall} \to \tilde{weight}$) $= \{0.81/x_1 + 0.64/x_2 + 0.49/x_3 + 0.16/x_4 + 0.04/x_5, 0.25/x_1 + 0.16/x_2 + 0.09/x_3 + 0.04/x_4 + 0.01/x_5\}.$

Fuzzy quantifiers

The fuzzy propositions may contain quantifiers like "very" , "more or less" etc. These fuzzy quantifiers may be eliminated as

Concentration

x is very $\tilde{A}$

$\mu_{very\ \tilde{A}}(x)=\{\mu_{very\,A}^{Belief}(x)^2, \mu_{very\,A}^{Disbelief}(x)^2\}$

x is very $\tilde{tall}$

$\mu_{very\ \tilde{tall}}(x) = \{0.81/x_1 + 0.64/x_2 + 0.49/x_3 + 0.16/x_4 + 0.04/x_5, 0.25/x_1 + 0.16/x_2 + 0.09/x_3 + 0.04/x_4 + 0.01/x_5\}.$

Diffusion

if x is more or less $\tilde{A}$
$\mu_{more\ or\ less\ \ \tilde{A}}(x)$=$\{\mu^{Belief}_{more\ or\ less\ A}(x)^2, \mu^{Disbelief}_{more\ or\ less\ A}(x)^{0.5}\}$
if x is more or less $t\tilde{a}ll$

$$\mu_{more\ or\ less\ \ t\tilde{a}ll}(x) = \{0.95/x_1 + 0.89/x_2 + 0.84/x_3 + 0.63/x_4 + 0.45/x_5,\\ 0.70/x_1 + 0.63/x_2 + 0.054/x_3 + 0.44/x_4 + 0.31/x_5\}.$$

3.3. The Fuzzy Reasoning with "Two Fold Fuzzy Sets"

The fuzzy reasoning is drawing a conclusions from fuzzy propositions.The fuzzy reasoning is given below for the fuzzy set with two fold membership functions and it is extension of fuzzy reasoning with single membership function.

R-1
x is $\tilde{A}$ and y is $\tilde{B}$
x is $\tilde{A_1}$

---

y is very $\tilde{A_1}$ o $(\tilde{A} \wedge \tilde{B})$
Rama is tãll and Sita is smãll
Rama is very tãll

Sita is very tãll o (tãll ∧ smãll)
tall = $\{0.8, 0.4\}$
more or less small = $\{0.77, 0.31\}$
very tall = $\{0.64, 0.16\}$
small = $\{0.6, 0.1\}$
very tãll o (tãll ∧ smãll)= $\{0.64, 0.16\}$ o $\{0.6, 0.1\}$ = $\{0.6, 0.1\}$

R-2
x is $\tilde{A}$ or y is $\tilde{B}$
x is $\tilde{A_1}$

---

y is very $\tilde{A_1}$ o $(\tilde{A} \vee \tilde{B})$
Rama is tãll or Sita is smãll
Rama is very tãll

Sita is very tãll o (tãll ∨ smãll)
very tãll o (tãll ∨ smãll)= $\{0.64, 0.16\}$ o $\{0.8, 0.4\}$ = $\{0.64, 0.16\}$

R-3
x and y are $\tilde{B}$
y and z are $\tilde{B}$

---

x and z are is $\tilde{A}$ o $\tilde{B}$
Rama and Sita are more or less tãll
Sita and Lakhmana are approximately eqũal

Rama and Lakhmana are is more or less tall $\tilde{\text{tall}}$ o approximately $\tilde{\text{equal}}$
approximately equal = {0.9, 0.3}
more or less tall= {0.83, 0.63 }
more or less tall $\tilde{\text{tall}}$ o approximately $\tilde{\text{equal}}$
= {0.83, 0.63 } o {0.9, 0.3}={0.83, 0.3}.

R-4

if x is $\tilde{A}$ then y is $\tilde{B}$
x is $\tilde{A}_1$

---

y is very $\tilde{A}_1$ o $(\tilde{A} \rightarrow \tilde{B})$
if Rama is $\tilde{\text{tall}}$ then Sita is $\tilde{\text{small}}$
Rama is very $\tilde{\text{tall}}$

Rama is very $\tilde{\text{tall}}$ o ($\tilde{\text{tall}}$ → $\tilde{\text{small}}$)
tall = {0.8, 0.4}
very tall = {0.64, 0.16}
small = {0.6, 0.1}
very $\tilde{\text{tall}}$ o ($\tilde{\text{tall}}$ → $\tilde{\text{small}}$)= {0.64, 0.16} o {0.8, 0.7} = {0.64, 0.16}

R-5

if x is $\tilde{A}$ then y is $\tilde{B}$ else y is $\tilde{C}$
x is $\tilde{A}_1$

---

y is very $\tilde{A}_1$ o $((\tilde{A} \times \tilde{B}) + (\tilde{A}' \times \tilde{C}))$
if Rama is $\tilde{\text{tall}}$ then Sita is $\tilde{\text{small}}$ else Sita is more or less $\tilde{\text{small}}$
Rama is very $\tilde{\text{tall}}$

Sita is very $\tilde{\text{tall}}$ o (($\tilde{\text{tall}}$ × $\tilde{\text{small}}$) + ($\tilde{\text{tall}}$’ × more or less $\tilde{small}$))
= {0.64, 0.16} o { min((0.6, 0.1), (0.2, 0.31)) }
={0.64, 0.16} o { 0.2, 0.1 }.
= { 0.2, 0.1}

R-6

if x is $\tilde{A}$ then y is $\tilde{B}$ else y is $\tilde{C}$
x is $\tilde{A}_1$

---

y is $\tilde{A}$ o $(\tilde{A}_1 \rightarrow \tilde{B})$
if Rama is $\tilde{\text{tall}}$ then Sita is $\tilde{\text{small}}$ else Sita is more or less $\tilde{\text{small}}$
Rama is very $\tilde{\text{tall}}$

Sita is very $\tilde{\text{tall}}$ o ($\tilde{\text{tall}}$ → $\tilde{\text{small}}$)
very $\tilde{\text{tall}}$ o ($\tilde{\text{tall}}$ → $\tilde{\text{small}}$)
= {0.64, 0.16} o{ (0.8, 0.7) }
= { 0.6, 0.16}.

R-7
if x is $\tilde{A}$ then y is $\tilde{B}$ else y is $\tilde{C}$
x is not $\tilde{A}_1$

---

y is not $\tilde{A}_1$ o $(\tilde{A}' \rightarrow \tilde{C})$
if Rama is tall then Sita is small else Sita is more or less small
Rama is not very tall

Sita is not very tall o (tall' → more or less small)
more or less small = {0.77, 0.31}
not very tall o (tall' → more or less small)
= {0.36, 0.84} o { 1, 0.71 } = { 0.36, 0.71}.

4. Fuzzy Certainty Factor

The fuzzy certainty factor(FCF) shall made as single fuzzy membership functions with two fuzzy membership functions to eliminate the conflict of evidence between "Belief"and "Disbelief".

Definition 4.1 *The FCF for propositions "x is $\tilde{A}$" is defined by its membership function $\mu_{\tilde{A}}^{FCF}(x) \rightarrow [0, 1]$, where $\mu_{\tilde{A}}^{FCF}(x)$ is single fuzzy set with difference between two fuzzy membership functions "Belief"and "Disbelief",*

$$\mu_{\tilde{A}}^{FCF}(x) = \{\mu_{A}^{Belief}(x) - \mu_{A}^{Disbelief}(x)\}/x,$$

$$\mu_{\tilde{A}}^{F}CF(x) < 0, \mu_{\tilde{A}}^{F}CF(x) = 0 \ \ and \ \ \mu_{\tilde{A}}^{F}CF(x) > 0$$

are the redundant, insufficient and sufficient respectively.

The FCF will compute the conflict of evidence of the incomplete information.
For Example
$\mu_{young}^{Belief}(x) = ((1 + (n/30)^2)^{-1}$
={0.9/10+0.8/15+0.69/20+0.59/25+0.5/30+0.42/35+0.36/40+0.31/45+0.26/50}
$\mu_{young}^{Disbelief}(x) = ((1 + ((n + 30)/30)^2)^{-1}$
={0.9/10+0.8/15+0.69/20+0.59/25+0.5/30+0.42/35+0.36/40+0.31/45+0.26/50 }
The Graphical representation of FCF is shown in Fig. 3.

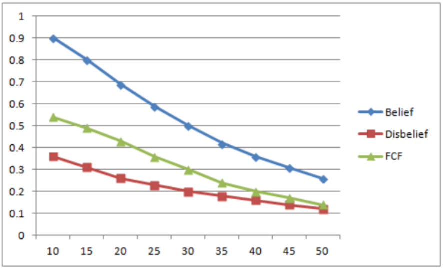

Fig.3 Fuzzy certainty factor

5. Fuzzy Truth Variables

Zadeh [18] defined quantification of truth variables as composition of fuzzy set and truth variables.

Definition 5.1 *The quantification of fuzzy truth variables for fuzzy set of fuzzy proposition of the type "x is A is* τ*" is defined as* $\mu_A^{-1}(x)$ *o* τ*,where* $\mu_A(x)^{-1}$ *is inverse of comparability function of A , "o" is composition and* τ *is fuzzy truth variable like true, false, very true etc.*

Definition 5.2 *The quantification of fuzzy truth variables for "a two fold fuzzy set" of fuzzy proposition of the type "x is* $\tilde{A}$ *is* τ*" may be defined as* $\mu_{\tilde{A}}(x)$ *o* τ*, where* $\mu_{\tilde{A}}(x)$ *is two fold fuzzy membership function.*

The truth functional modification of fuzzy proposition "x is $\tilde{A}$ is very true" is given

$\{\mu_A^{Belief}(x), \mu_A^{Disbelief}(x)\}$ o very true=$\{\mu_{very\,A}^{Belief}(x), \mu_A^{Disbelief}(x)\}$

The truth functional modification of fuzzy proposition "x is $\tilde{A}$ is very false"is given

$\{\mu_A^{Belief}(x), \mu_A^{Disbelief}(x)\}$ o very false =$\{\mu_A^{Belief}(x), \mu_{very\,A}^{Disbelief}(x)\}$,
"true" is applied on Belief and "false"is applied on Disbelief for quantification of truth variables.

The truth functional modification of fuzzy proposition "tallness of x is very true" is given as

$$\tilde{\text{tall}} = \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5,$$
$$0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\},$$
$$\text{very}\,\tilde{\text{tall}} = \{0.81/x_1 + 0.64/x_2 + 0.49/x_3 + 0.16/x_4 + 0.04/x_5,$$
$$0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}.$$

The truth functional modification of fuzzy proposition "tallness of x is very false" is given as

$$\tilde{\text{tall}} = \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5,$$
$$0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\},$$
indent $\text{very}\,\tilde{\text{tall}} = \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5,$
$$0.25/x_1 + 0.16/x_2 + 0.09/x_3 + 0.04/x_4 + 0.01/x_5\}.$$

The nested fuzzy propositions of the form

x is $\tilde{A}$ is $(\tau_1$ is $(\tau_2 \cdots$ is $\tau_n)) = x$ is $\tilde{A}$ o $\tau_1)$ o τ_2 o $\cdots$ o τ_n.

6. Comparison with Fuzzy Set and "Two Fold Fuzzy Sets"

The fuzzy set defined with two membership functions will give more information than a fuzzy set with single membership function. The "two fold fuzzy sets" is a class of objects with a continuum of two grades of memberships "Belief"and "Disbelief".

$\tilde{A}= \{\mu_A^{Belief}(x), \mu_A^{Disbelief}(x)\}$,
$\mu_A^{Belief}(x) = \mu_A^{Belief}(x_1)/x_1 + \cdots + \mu_A^{Belief}(x_n)/x_n$,
$\mu_A^{Disbelief}(x)= \mu_A^{disbelief}(x_1)/x_1 + \cdots + \mu_A^{Disbelief}(x_n)/x_n$, where "+" is union.

The fuzzy set with single membership function is defined as class of objects with continuum of single grade function $A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \mu_A(x_n)/x_n$.

The two opinions are better judge than single opinion.

The fuzzy proposition “x is Headache”is given with single membership function.

For instance the fuzziness of “Rama who is 40 years old is having Headache is 0.6”

$$\mu_{Headache}(x) = \{0.2/x_1 + 0.35/x_2 + 0.55/x_3 + 0.6/x_4 + 0.65/x_5 + 0.7/x_6 + 0.75/x_7 + 0.85/x_8 + 0.9/x_9\}$$

The fuzzy proposition “x is Headache” is given with two membership functions.

For instance the fuzziness of “Rama who is 40 years old is having Headache is $\{0.65, 0.16\}$,”

$$\mu^{Belief}_{Headache}(x) = \{0.25/x_1 + 0.4/x_2 + 0.6/x_3 + 0.65/x_4 + 0.7/x_5 + 0.75/x_6 + 0.85/x_7 + 0.9/x_8 + 0.95/x_9\}$$

$$\mu^{Disbelief}_{Headache}(x) = \{0.1/x_1 + 0.12/x_2 + 0.14/x_3 + 0.16/x_4 + 0.2/x_5 + 0.24/x_6 + 0.28/x_7 + 0.3/x_8 + 0.32/x_9\}$$

The Graphical representation of two fold sets of Headache “belief” and “Disbelief” are shown in Fig. 4.

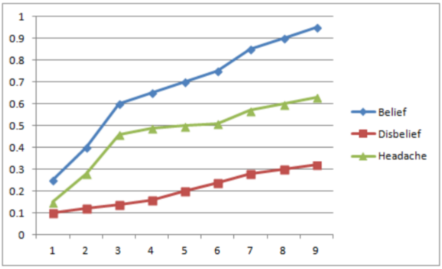

Fig.4 Two fold fuzzy membership function of headache

$$\mu^{FCF}_{\tilde{A}}(x) = \{\mu^{Belief}_{A}(x) - \mu^{Disbelief}_{A}(x)\},$$

For instance ” the Patient has Headache” with FCF is given as

$$\mu^{FCF}_{\tilde{A}}(x) = \mu^{Belief}_{A}(x) - \mu^{Disbelief}_{A}(x) = \{0.15/x_1 + 0.28/x_2 + 0.46/x_3 + 0.49/x_4 + 0.5/x_5 + 0.51/x_6 + 0.57/x_7 + 0.6/x_8 + 0.63/x_9\}$$

The FCF is defined as single membership function with the difference between “Belief” and “Disbelief”fuzzy membership functions.

The Graphical representation of “Headache” with single membership function “Belief” and “FCF” of Headache are shown in Fig. 5.

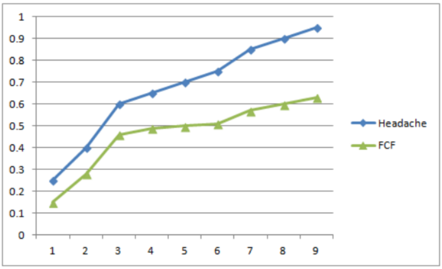

Fig.5 Two fold fuzzy set membership functions headache

7. Fuzzy Medical Diagnosis as Application of "Two Fold Fuzzy Sets"

Usually medical diagnosis is made on the basis of belief. EMYCIN [1] deals incomplete information with Probability. The fuzzy logic deal with belief rather than probability. EMYCIN is medical expert system shell (empty knowledge base) in which specialized medical expertise is defined. The knowledge engineering in expert system consist of knowledge base (medical rule in the form of "if A then B", knowledge acquisition (extracting knowledge from domain expert), inference methods(forward and backward mechanism), inference (reasoning) and explanation facility.

EMYCIN is Medical expert system shell in which the incomplete information is defied as MB[h,e] and MD[h,e], where "h" is hypothesis and "e" is evidence.

Buchanan and Sortliffe [1] defined MB[h,e] and MD[h,e] as probability functions with belief and disbelief. where "e" is evidence for given hypothesis "h" in MYCIN. The certainty factor is defines as $CF[h,e] = MB[h,e] - MD[h,e]$, where MB[h,e] and MD[h,e] probabilities.

The fuzzy function is considered instead of probability.

$MB[x,A] = MB[h,e] = \mu_A^{Belief}(x)$,

$MD[x,A] = MD[h,e] = \mu_A^{Disbelief}(x)$,

where MB[x,A] and MD[x,A] are fuzzy functions.

$MB[x,A] = MB[h,e] = \int MB[h,e]\mu_A^{Belief}(x)/x$,

$MD[x,A] = MD[h,e] = \int MB[h,e]\mu_A^{Disbelief}(x)/x$,

where MB[x,A] and MD[x,A] are fuzzy probabilities.

The fuzzy certainty factor(FCF) is given for the proposition of the type "x is $\tilde{A}$" as $\mu_{\tilde{A}}^{FCF}(x) = \mu_A^{Belief}(x) - \mu_A^{Disbelief}(x)$.

For instance "the Patient has fever"

$\mu_{fever}^{FCF}(x) = \mu_{fever}^{Belief}(x) - \mu_{fever}^{Disbelief}(x)$.

Consider the fuzzy rule in medical diagnosis.

if the patient has fever
and patient has rash
and patient has body-ache
and patient has chills
then the patient has chicken-pox.

Using fuzzy modulations/ Fuzzy knowledge representation the above rule may be represented as [15]

if symptom (patient, fever)
and symptom (patient, rash)
and symptom (patient, body-ache)
and symptom (patient, chills)
then symptom (patient, chicken-pox).

In Prolog, it may be define as

symptom(patient, chickenpox) :-symptom(patient, fever),
symptom(patient, rash), symptom(patient, body ache), symptom(patient, chills).

For instance, fuzziness is give for symptoms and diagnosis as

if the patient has fever (0.9, 0.2)
and patient has rash (0.8, 0.2)

and patient has body ache (0.7, 0.1)
and patient has chills (0.9, 0.1)
and the patient has chicken-pox (0.7, 0.2).
The FCF are given as
if the patient has fever (0.7)
and patient has rash (0.6)
and patient has body-ache (0.6)
and patient has chills (0.8)
then the patient has chicken-pox (0.5).
The Zadeh fuzzy conditional inference is given as
if x is $\tilde{A}_1$ and x is $\tilde{A}_2$ and $\cdots$ and x is $\tilde{A}_n$ then y is $\tilde{B}$
$= \min (1, 1\text{-}(\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n + \tilde{B}_n)$
if FCF1 and FCF2 and FCF3 and FCF4 then FCF5
= min(1, 1 − (min(FCF1, FCF2, FCF3, FCF4) + FCF5).
Using LISP language, the FCF may be computed for the above rule as

```
(defun fcf (fcf1 fcf2 fcf3 fcf4 fcf5)
( min 1 ( + (-1 (min fcf1 fcf2 fcf3 fcf4)) fcf5))).
FCF
(fcf 0.7 0.6 0.6 0.8 0.5)
0.9
```

The above rule interpret in EMYCIN as

```
IF (AND
(SAME CNTEXT FEVER)
(SAME CNTEXT RASH)
(SAME CNTEXT BODY-ACHE)
(SAME CNTEXT CHILLS)
THEN (CONCLUDE CNTEXT CHICKEN-POX TALLY 0.9).
```

The EMYCIN will diagnose chicken-pox with fuzzy certainty factor 0.9.
The Mamdani fuzzy conditional inference is given by
if x is $\tilde{A}_1$ and x is $\tilde{A}_2$ and $\cdots$ and x is $\tilde{A}_n$ then y is $\tilde{B}$
$= \min (\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n, \tilde{B})$
if FCF1 and FCF2 and FCF3 and FCF4 then FCF5
= min(FCF1,FCF2,FCF3,FCF4, FCF5)
Using LISP language , the FCF may be computed for the above rule as

```
(defun fcf (fcf1 fcf2 fcf3 fcf4 fcf5)
(min fcf1 fcf2 fcf3 fcf4 fcf5))
FCF
(fcf 0.7 0.6 0.6 0.8 0.5)
0.5
```

The above rule interpret in EMYCIN as

```
IF (AND
(SAME CNTEXT FEVER)
(SAME CNTEXT RASH)
(SAME CNTEXT BODY-ACHE)
(SAME CNTEXT CHILLS)
```

THEN (CONCLUDE CNTEXT CHICKEN-POX TALLY 0.5)
The EMYCIN will diagnose chicken-pox with fuzzy certainty factor 0.5.

Usually in medical diagnosis prior information is not available to the consequent part i.e., the diagnosis will be made from symptoms. In such a case the following fuzzy inference may be used

The Reddy fuzzy conditional inference is given by

if x is $\tilde{A}_1$ and x is $\tilde{A}_2$ and $\cdots$ and x is $\tilde{A}_n$ then y is $\tilde{B}$

$= \min(\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n)$

if FCF1 and FCF2 and FCF3 and FCF4 then FCF5 =min (FCF1,FCF2,FCF3,FCF4).

Using LISP language , the FCF may be computed for the above rule as

```
(defun fcf (fcf1 fcf2 fcf3 fcf4)
(min fcf1 fcf2 fcf3 fcf4))
FCF
(fcf 0.7 0.6 0.6 0.8)
0.6.
```

The above rule is interpretd in EMYCIN as

IF (AND
(SAME CNTEXT FEVER)
(SAME CNTEXT RASH)
(SAME CNTEXT BODY-ACHE)
(SAME CNTEXT CHILLS)
THEN (CONCLUDE CNTEXT CHICKEN-POX TALLY 0.6).

The EMYCIN will diagnose chicken-pox with fuzzy certainty factor 0.6.

8. Conclusion

Various theories are proposed to deal with incomplete information. Many theories deal with Probability (likelihood), where as fuzzy logic deals with commonsense. Zadeh first proposed fuzzy with single membership function for incomplete information then many Researchers extended his work. The fuzzy set with the two fold fuzzy membership function will give more evidence than a single fuzzy membership one. The fuzzy logic with two fold fuzzy membership function is studied based on “Belief”and “Disbelief”. The operations on fuzzy sets with two fold fuzzy membership functions are studied. The fuzzy Inference and fuzzy reasoning are studied for “a two fold fuzzy sets”. The FCF is studied as the difference between the two fuzzy membership functions “Belief”and “Disbelief”. The fuzzy Certainty Factor is made as a single fuzzy membership function to compute the conflict of evidence of the Incomplete Information. The fuzzy truth variables are studied for “a two fold fuzzy set”. The fuzzy medical expert system is discussed as application for “a two fold fuzzy set”. Besides this works, there is possibility of extend this work to “three fold fuzzy sets”.

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